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Underwater Explosion Bubbles IV. Summary of Results and Numerical Computations

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AFSWP-1015

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UNDERWATER EXPLOSION BUBBLES IV. SUMMARY OF RESULTS AND NUMERICAL COMPUTATIONS.

by

Ignace I. Kolodner

AFSWP-1015

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PREFACE

In this concluding report of a series of four on "Underwater Explosion Bubbles" we present results of numerical computations for specific underwater explosions. The work on these computations began in 1953 and was completed, except for various drawings, in 1955.

While the organization of the work and the responsibility for its accuracy are assumed by the undersigned, actual work has been largely carried out by others. Professor E. Isaacson was frequently consulted on the computational techniques. Miss M. Reissman, Mrs. S. Hahn, L. D. Grey and Miss L. Wertheimer did all the computations. Drawings were prepared by Mrs. S. Hahn and Miss L. Wertheimer and tables were extracted by J. Smith and Miss E. Kramer.

July 1956

Ignace I. Kolodner

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I. Introduction.

In this report we shall discuss the shape and migration of an underwater bubble produced by an explosion of 300 lb. TNT fired 125 ft. below the sea level.[1] The numerical results were obtained on the basis of the theory developed in IMM-NYU Reports 197 and 232 on Underwater Explosion Bubbles, see [3], [4] .

Two sets of results are presented. The first refers to a bubble in infinite water. The second - to a bubble fired 20 ft. above the sea bottom. Most of the computations can be reused for description of other cases having the same value of the parameter k . For definition of this and other parameters, see Section III.

Our results are only partly satisfactory. They are in qualitative and even fair quantitative agreement with observed data for a brief period of time. Yet we lack a mathematical criterion to determine the time at which they cease to represent a good approximation even to the idealized problem posed under the assumptions of Section II. Thus, when considering the bubble motion in infinite water, we find that, to the order of approximation used, the bubble begins to move downwards after a certain time, and this is absurd in the case considered. Next we find that the assumed expansion for the water velocity potential becomes meaningless at the time when the center of gravity of the bubble reaches its surface. This happens near the end of the first oscillation period at which time the bubble

---

1 These data are typical for explosions of "moderate" size. They were suggested by Dr. A. B. Arons.

has a fully developed cavity, and its shape indicates a possible change into toroidal shape. However, it is expected in this case that the bubble will perform several oscillations before changing its topological structure. Altogether we found it difficult to use the numerical results for more than one oscillation period. In order to obtain a description of the bubble motion for two periods, we had to reduce the value of the parameter σ to .00125, and this corresponds to an explosion of 1.05 ~~grams~~ (t) of TNT at a depth of 125 ft.

II. Mathematical Formulation.

Our study is based on the following simplifying assumptions.

1. The ocean water is incompressible and inviscid.
2. The water flow is laminar and irrotational.
3. The bubble gas has no internal motion and expands adiabatically.
4. The bubble is initially spherical and has no radial velocity.
5. The ocean bottom is a rigid surface.
6. The bubble is sufficiently far from the water surface to allow the use of a linearized surface condition.

The object is to find the water velocity potential, $\Phi(x,y,z,t)$, the migration of the center of gravity of the bubble, B(t), measured from the center of explosion and the equation of the bubble surface, $r = R(\theta,t)$. The equation for the bubble surface has been written in spherical coordinates with origin at the moving center of gravity

of the bubble, and the polar axis pointing upwards.

Under the above assumptions, Φ, B, and R are found to satisfy and to be determined by the following conditions:

(2.1) $\Delta \Phi = 0$, in water,

(2.2) $\nabla \Phi \cdot \nabla F + F_t = 0$, for $F = 0$ (kinematic bubble surface condition),

(2.3) $P_o - \rho[\Phi_t + \frac{1}{2}(\nabla\Phi)^2] - \rho g z = KV^{-\gamma}$, for $F = 0$ (dynamic bubble surface condition),

(2.4) $\Phi_z = 0$, for $z = -H$ (rigid bottom condition),

(2.5) $\Phi = 0$, for $z = z_o$ (linearized free surface condition),

(2.6) $R(\theta,0) = A_o$ = constant,

(2.7) $R_t(\theta,0) = 0$,

(2.8) $B(0) = \dot{B}(0) = 0$,

(initial conditions).

Here,

ρ - density of water

p_o - atmospheric pressure

z_o - depth of explosion center

H - distance of the bottom from the explosion center

$P_o = p_o + \rho g z_o$ - hydrostatic pressure at the explosion center

$$F(x,y,z,t) = z - B(t) - R(\theta,t)\cos\theta \ , \ \cos\theta = \frac{z - B}{\sqrt{x^2+y^2+(z-B)^2}} \ ,$$

$$V = \frac{2\pi}{3}\int_0^{\pi} R^3(\theta,t)\sin\theta \, d\theta = \text{volume of the bubble.}$$

γ - adiabatic exponent.

$\varkappa$ - adiabatic gas constant

W - weight of the explosive

$K = \varkappa W^{\gamma}$.

A_0 - initial radius of the bubble.

When the effects of the boundaries can be neglected, the conditions (2.4,5) are replaced by a requirement of regularity of Φ . One obtains the same result by solving the problem with finite H and z_0, and letting $H \longrightarrow \infty$, $z_0 \longrightarrow \infty$.

III. Dimensional Analysis.

The number of parameters appearing in the formulation is reduced by introducing dimensionless units and parameters. It is convenient to introduce for units of length L and time T the following:

$$L = \left(\frac{3(\gamma-1)E}{4\pi\gamma P_0}\right)^{1/3} \ , \tag{3.1}$$

$$T = L\sqrt{\rho P_0^{-1}} \ . \tag{3.2}$$

Here,

$$E = \frac{4\pi}{3} A_o^3 [\tfrac{3}{2}\rho \dot{A}_o^2 + P_o + \frac{K}{\gamma-1}(\frac{4\pi}{3} A_o^3)^{-\gamma}] \quad , \tag{3.3}$$

is the total constant energy that the system water bubble would have in the absence of boundaries. When boundaries are present, the total energy is still constant but different from E by a term $\overline{E}$. However, one shows that even in this case $\overline{E} = 0$ if the initial radial velocity of the bubble, $\dot{A}_o = 0$.

The units used here are different from the units introduced by Friedman [2], and adopted in the literature on Underwater Explosion Bubbles. If $\overline{L}$ and $\overline{T}$ are the Friedman units, we have the relations

$$\begin{cases} L = (\frac{\gamma-1}{\gamma})^{1/3}\overline{L} \quad , \\ T = \sqrt{\frac{3}{2}}(\frac{\gamma-1}{\gamma})^{1/3}\ \overline{T} \ . \end{cases} \tag{3.4}$$

For $\gamma = 1.25$, a value generally accepted for explosion products, see [1],

$$\begin{aligned} L &= .585\ \overline{L} \quad , \\ T &= .716\ \overline{T} \quad . \end{aligned} \tag{3.4'}$$

L is the equilibrium radius of a bubble in a system of energy E, while L' has no exact physical meaning, though it is close to and always somewhat less than the theoretical maximum radius of the bubble. The choice of T is motivated by convenience.

On introducing dimensionless variables, four independent parameters appear, namely:

(3.5) $$\varkappa = \frac{K}{\gamma-1} P_o^{\gamma-1} E^{-\gamma} ,$$

(3.6) $$\sigma = \frac{L}{Z_o} ,$$

(3.7) $$\nu = \frac{H}{Z_o} ,$$

(3.8) $$\mu = \frac{z_o}{Z_o} .$$

Here, $Z_o = z_o + p_o/\rho g$, is the hydrostatic depth ("head") of the explosion center.

In a scaled experiment, the values of $\varkappa$, σ, μ, and ν must be preserved. Using (3.3) in (3.5), and assuming $\dot{A}_o = 0$, (see assumption 4, Section II) one can show that

(3.9) $$\varkappa = x(1 + x)^{-\gamma} ,$$

where

(3.10) $$x = \frac{P_m}{P_o(\gamma-1)} .$$

Here, P_m is the maximum pressure of the bubble which depends only on the type of explosive, but not on its amount. See also [7], p. 4. Using the same notation,

(3.11) $$E = P_o V_o (1 + x) .$$

Even for large values of z_o, the internal energy of the bubble is large compared to its potential energy. This is equivalent to the assumption $x >> 1$, whence the simplified approximate formulae

(3.9') $$\varkappa \sim x^{1-\gamma} \propto Z_o^{\gamma-1}$$

(3.10') $$E \sim P_o V_o x = \varepsilon W \quad ,$$

where ε is the specific energy of the bubble.

Formula (3.9) shows that a scaled experiment with the same explosive is impossible. On the other hand, it is possible to produce scaled experiments with different types of explosives, provided that the pressure above the free surface is reduced. Denote by prime the quantities referring to a scaled experiment. Let $P_m' = pP_m$, $\varepsilon' = q\varepsilon$, $\rho' = r\rho$. Then the choice: $p_o' = pp_o$, $z_o' = pr^{-1}z_o$, $H' = pr^{-1}H$, and $W' = p^4q^{-1}r^{-3}W$ will preserve the values of all the parameters, while $L' = pr^{-1}L$, $T' = \sqrt{pr^{-1}}\ T$. See [5], p. 11.

IV. Formulae for Bubble, Migration, and Potential Coefficients.

We define $\phi(\overline{r},\theta,\overline{t})$, $\lambda(\theta,\overline{t})$, $b(\overline{t})$ by

(4.1) $$B(t) = Lb(\tfrac{t}{T})$$

(4.2) $$R(\theta,t) = L\lambda(\theta, \tfrac{t}{T})$$

(4.3) $$\Phi(x,y,z,t) = L^2T^{-1}[\dot{b}(\tfrac{t}{T})\ \tfrac{z}{\lambda} + \phi(\tfrac{r}{L}\ ,\ \theta,\ \tfrac{t}{T})] \quad .$$

One shows in [3] and [4] that ϕ, λ, and b, can, on dropping all bars, be represented in the form:

$$(4.4)\qquad \phi = \sum_{n=0}^{\infty} \sigma^n \sum_{m=0}^{n} c_{nm}(t)(r^{-(m+1)}P_m(\cos\theta) + \phi_m^*(r,\theta))$$

$$(4.5)\qquad b = \sum_{n=1}^{\infty} \sigma^n b_n(t)$$

$$(4.6)\qquad \lambda(\theta,t) = \sum_{n=0}^{\infty} \sigma^n \sum_{m=0}^{n} a_{nm}(t)P_m(\cos\theta) .$$

Here, $P_m(\cos\theta)$ is the m'th Legendre polynomial. For definition of ϕ_m^* see [4], Appendix A.

The coefficients a_{nm}, b_n and c_{nm} are defined, depending on the case, by either explicit formulae, or integrals, or as solutions of ordinary differential equations. To list them it is convenient to introduce first the following functions and notations:

1) The function $\alpha(\tau)$ is an even periodic function defined for half a period by its principal inverse, $\tau(\alpha)$ given by

$$\tau(\alpha) = \int_{\bar{\alpha}}^{\alpha} \frac{dx}{\sqrt{x^{-3}-1-\varkappa x^{-3\gamma}}} , \quad \underline{\alpha} \leq \alpha \leq \bar{\alpha} .$$

Here $\underline{\alpha}$ and $\bar{\alpha}$ are respectively the smallest and largest roots of

$$1 - x^3 - \varkappa x^{-3(\gamma-1)} = 0 .$$

2) $f(x) = 2\frac{d}{dx}[\log \Gamma(\frac{1+x}{4}) + \log \Gamma(\frac{1-x}{4})] - \frac{\pi}{2}\tan\frac{\pi x}{2}$, $|x| \leq 1$

3) $x = \frac{\nu-\mu}{\nu+\mu}$

4) $d_{oo1} = -(\mu+\nu)^{-1}(f(x) + \log 2)$

$d_{o12} = -(\mu+\nu)^{-2}f'(x)$

$d_{o23} = -(\mu+\nu)^{-3}(\frac{1}{2}f''(x) + \frac{3}{16}\sum_{s=1}^{\infty} s^{-3})$.

The functions $f(x)$ and $f'(x)$ are tabulated in [2], p. 59 and p. 63. For convenience, we tabulate here again $f(x)$, $f'(x)$, and in addition $f''(x)$, for $0 \leq x < 1$. For negative values of x, $f(x) = -f(-x)$, $f'(x) = f'(-x)$, and $f''(x) = -f''(-x)$. See Table 1.

5) For any function $h(t)$, we use the notation,

$$\bar{h}(t) = \int_0^t h(\tau)d\tau \quad .$$

We now list the a's, b's and c's up to terms in σ^3 .

Zero order terms.

$$a_{oo}(t) = a(t) = (\frac{\gamma}{\gamma-1})^{1/3}\alpha(\sqrt{\frac{3}{2}}(\frac{\gamma-1}{\gamma})^{1/3}t) \quad ,$$

$$c_{oo}(t) = -a^2\dot{a} \quad .$$

First order terms

$$a_{1o} = -\frac{1}{2}d_{oo1}\dot{a}\bar{a} = d_{oo1}\alpha_{1o}$$

$$a_{11} \equiv 0$$

$$b_1 = 2\ \overline{a^{-3}\overline{a^3}}$$

$$c_{1o} = -\ (a^2 \dot{a}_{1o})^{\cdot}$$

$$c_{11} = -\ \overline{a^3}\ .$$

<u>Second order terms</u>

$$a_{2o} = \tilde{a}_{2o} + d_{oo1}^2 \alpha_{2o}\ ,$$

where

$$a\ddot{\tilde{a}}_{2o} + 3\dot{a}\dot{\tilde{a}}_{2o} + [\ddot{a} + 3\gamma^2(\frac{\gamma}{\gamma-1})^{\gamma-1}\dot{x}a^{-3\gamma-1}]\tilde{a}_{2o} = b_1 + \frac{1}{4}\dot{b}_1^2\ ,$$

$$\bar{a}_{2o}(0) = \dot{\bar{a}}_{2o}(0) = 0\ ,$$

$$\alpha_{2o} = \frac{1}{8}[\ddot{a}(\bar{a})^2 + 2a\dot{a}\bar{a} + \dot{a}\overline{a^2}\ ]\ .$$

$$a_{21} \equiv 0$$

a_{22} satisfies

$$a\ddot{a}_{22} + 3\dot{a}\dot{a}_{22} - \ddot{a}a_{22} = -\ \frac{9}{4}\ \dot{b}_1^2\ , \quad a_{22}(0) = \dot{a}_{22}(0) = 0\ .$$

$$b_2 = d_{oo1}\beta_2 + d_{o12}\beta_2'$$

where

$$(a^3\dot\beta_2)^{\bullet} = \frac{3}{2}\, a\dot b_1(\ddot a a\bar a + a^2\dot a - \dot a^2\bar a)\ , \qquad \beta_2(0) = \dot\beta_2(0) = 0\ ,$$

$$(a^3\dot\beta_2')^{\bullet} = -\,3a^2(a^3\dot a)^{\bullet}\ , \qquad \beta_2'(0) = \dot\beta_2'(0) = 0\ .$$

$$c_{20} = -\,(a^2a_{20} + aa_{10}^2)^{\bullet}$$

$$c_{21} = -\,\frac{1}{2}(a^3\dot b_2 + 3a^2a_{10}\dot b_1 + d_{012}a^5\dot a)$$

$$c_{22} = -\,\frac{1}{3}\, a^2(a^2a_{22})^{\bullet}\ .$$

Third order terms.

$$a_{30} = d_{001}\alpha_{30} + d_{012}\alpha_{30}' + d_{001}^3\alpha_{30}''\ ,$$

where,

$$\ddot\alpha_{30} - \left(\frac{\dddot a}{\dot a}\right)\alpha_{30} = -\,\frac{1}{2}\left[\left(\frac{\dddot a}{\dot a}\right)^{\bullet}\bar a + 2a\left(\frac{\dddot a}{\dot a}\right) + 4\ddot a\right]\tilde a_{20} - \frac{1}{3}\, b_1$$

$$+\,\frac{1}{4}\,\dot b_1^2(a^{-2}\dot a\bar a - \frac{5}{3}) + a^{-1}\dot b_1\dot\beta_2\ , \quad \alpha_{30}(0) = \dot\alpha_{30}(0) =$$

$$\dot\alpha_{30}' - \left(\frac{\dddot a}{\dot a}\right)\alpha_{30}' = -\,(2a\ddot a + \dot a^2)b_1 + a\dot a\dot b_1 - a^2 - a\dot b_1\dot\beta_2'\ ,$$

$$\alpha_{30}'(0) = \dot\alpha_{30}'(0) = 0\ ,$$

$$\alpha_{30}'' = -\,\frac{1}{16}\left[\frac{1}{3}\,\dddot a(\bar a)^3 + \ddot a\bar a\,\overline{a^2} + (2a\ddot a + \dot a^2)(\bar a)^2 + 3a^2\dot a\bar a\right.$$

$$\left. +\, a\dot a\overline{a^2} + \dot a\overline{a^3}\right]\ .$$

$$a_{31} \equiv 0\ ,$$

$$a_{32} = d_{oo1}\alpha_{32} + d_{o12}\alpha'_{32} + d_{o23}\alpha''_{32} \quad ,$$

where

$$a\ddot{\alpha}_{32} + 3\dot{a}\dot{\alpha}_{32} - \ddot{a}\alpha_{32} = -\frac{9}{2}\dot{b}_1\dot{\beta}_2 - (\alpha_{1o}\ddot{a}_{22} + 3\dot{\alpha}_{1o}\dot{a}_{22} - \ddot{\alpha}_{1o}a_{22}) \; ,$$

$$\alpha_{32}(0) = \dot{\alpha}_{32}(0) = 0 \quad ,$$

$$a\ddot{\alpha}'_{32} + 3\dot{a}\dot{\alpha}'_{32} - \ddot{a}\alpha'_{32} = -\frac{9}{2}\dot{b}_1(\dot{\beta}'_2 + a^2\dot{a}) \; , \quad \alpha'_{32}(0) = \dot{\alpha}'_{32}(0) = 0 \; ,$$

$$a\ddot{\alpha}''_{32} + 3\dot{a}\dot{\alpha}''_{32} - \ddot{a}\alpha''_{32} = -5(a^4\dot{a})^{\cdot} \; , \quad \alpha''_{32}(0) = \dot{\alpha}''_{32}(0) = 0 \; .$$

a_{33} satisfies

$$a\ddot{a}_{33} + 3\dot{a}\dot{a}_{33} - 2\ddot{a}a_{33} = -6\dot{b}_1(\dot{a}_{22} + \frac{1}{5}\dot{a}^{-1}\ddot{a}a_{22}), \; a_{33}(0) = \dot{a}_{33}(0) =$$

$$b_3 = \tilde{b}_3 + d_{oo1}^2\beta_3 + d_{oo1}d_{o12}\beta'_3 \quad ,$$

where

$$(a^3\dot{\tilde{b}}_3)^{\cdot} = -3(a^2\dot{b}_1\tilde{a}_{2o})^{\cdot} + 6a^2\tilde{a}_{2o} + \frac{9}{5}(a^2\dot{b}_1a_{22})^{\cdot}, \; \tilde{b}_3(0) = \dot{\tilde{b}}_3(0) =$$

$$(a^3\dot{\beta}_3)^{\cdot} = -3(\dot{\beta}_2a^2\alpha_{1o} + \dot{b}_1a\alpha_{1o}^2 + \dot{b}_1a^2\alpha_{2o})^{\cdot} + 6(a^2\alpha_{2o} + a\alpha_{1o}^2) \; ,$$

$$\beta_3(0) = \dot{\beta}_3(0) = 0 \; ,$$

$$(a^3\dot{\beta}'_3)^{\cdot} = -3(\dot{\beta}'_2a^2\alpha_{1o} + (a^5\alpha_{1o})^{\cdot})^{\cdot} + 12a^2\dot{a}(a^2\alpha_{1o})^{\cdot} \; ,$$

$$\beta'_3(0) = \dot{\beta}'_3(0) = 0 \; .$$

$$c_{3o} = -(a^2 a_{3o} + a a_{1o} a_{2o} + \frac{1}{3} a_{1o}^3)$$

$$c_{31} = -\frac{1}{2}[a^3 \dot{b}_3 + 3\dot{b}_2 a^2 a_{1o} + 3\dot{b}_1 a a_{1o}^2 + 3\dot{b}_1 a^2(\tilde{a}_{2o} - \frac{1}{5}a_{22}) + (a^5 a_{1o})^{\cdot} d_{o12}] \,.$$

The c's are needed for the study of the pressure field which is not discussed in this paper. c_{32} and c_{33} were not computed and are of no interest.

V. Choice of Parameters and Computational Procedure

To determine the values of units and parameters for the cases discussed in this report, we use the data for TNT supplied in [1]. These are

$$(5.1) \qquad \begin{cases} \gamma = 1.25 \\ \varepsilon = 490 \text{ cal/g.} \\ \varkappa = .055\, Z_o^{\gamma-1} \,, \quad Z_o \text{ expressed in feet .} \end{cases}$$

To determine $\varkappa$, one uses the simplified formula (3.9') instead of (3.9). This introduces an error of less than .2°/o in our case. Using (5.1) we get for a bubble produced by 300 lb. TNT fired at a depth of 125 ft. (158 ft. head):

$$E = 2.79 \times 10^{15} \text{ erg}$$
$$L = 10.00 \text{ ft.}$$
$$T = .1403 \text{ sec.}$$
$$\varkappa = .1957$$
$$\sigma = .0633$$

For a bubble in infinite water, $\mu = \infty$, $\nu = \infty$, $d_{oo1} = d_{o12} = d_{o23} = 0$, and the only quantities needed are: a, b_1, $\tilde{a}_{2o}$, a_{22} , $\tilde{b}_3$, and a_{33} . We consider also the case when the effect of boundaries is taken into account, and the bottom is 20 ft. below the explosion center. Then:

$$\mu = .791$$
$$\nu = .1266$$
$$x = \frac{\nu-\mu}{\nu+\mu} = -\ .724$$

and

$$d_{oo1} = 2.43$$
$$d_{o12} = -\ 15.85$$
$$d_{o23} = 61.8 \quad .$$

The last example considered is that with $\mu = \infty$, $\nu = \infty$, $\varkappa = .1957$ and $\sigma = .00125$. This corresponds to an explosion of 1.05 grams of TNT at 125 ft. The corresponding units are:

$$E = 2.15 \times 10^{10} \text{ erg.}$$
$$L = 2.37 \text{ in.}$$
$$T = .00277 \text{ sec.}$$

The functions, a, a_{1o}, b_1, $\tilde{a}_{2o}$, α_{2o}, β_2, β_2', α_{3o}, α_{3o}' , α_{3o}'' , α_{32} , α_{32}' , α_{32}'' , a_{33} , $\tilde{b}_3$, β_3, and β_3' depend on $\varkappa$, but not on the other parameters. They were computed for a single value of

$x = .1957$ over the time of two periods of $a(t)$, and are tabulated, together with $\dot{a}$, $\ddot{a}$, $\dot{b}_1$ in tables 2 - 6. While this computation presents many difficulties, all remaining computations involve only the operations of addition and multiplication, and can be completed in a relatively shorter time.

The difficulties encountered in the computations may be traced to the following:

1. Quantities with a given subscript depend on quantities with lower first subscripts, necessitating a very accurate computation of all quantities with a low first subscript. In particular, we found that $\underline{a(t)}$, on which all other quantities depend sensitively, must be computed accurately to 8 decimals in order to get about 2 relevant figures in the computation of, say, a_{33}.

2. All quantities vary very rapidly during a short interval near the time of minima of $a(t)$, necessitating frequent changes of intervals in any finite difference scheme. In particular, to compute $a(t)$ we had to start with an interval of .0004 at the minimum of a. This interval has been progressively increased to the value .1 near the maximum of a.

3. The differential equations defining these quantities are unstable and produce results which are either growing rapidly or oscillating rapidly, and in which errors become soon uncontrollable. For a study of these instabilities, see [6].

The function $\alpha(\tau)$ was computed by using a finite difference scheme. All coefficients defined by integrals, namely, α_{10}, b_1,

α_{2o} , β_2, β_2' , α_{3o}'' , β_3, β_3' were computed by using the trapezoidal rule, using intervals of varying length which will be discussed presently.

The coefficients $\tilde{a}_{2o}$, a_{22}, α_{3o}, α_{3o}', α_{32}, α_{32}', α_{32}', a_{33} are defined as solutions of non-homogeneous linear equations of second order. All these equations can be reduced to the form

$$\ddot{y}(t) + g(t)y(t) = h(t) \ .$$

for $y(t)$, where $g(t)$ depends only on $a(t)$ and its derivatives. (Of course, $g(t)$ and $f(t)$ vary from equation to equation.) Rather tha use a finite difference scheme, we assumed that $g(t)$ is piecewise constant and $h(t)$ piecewise linear during short time intervals Δt . These intervals were so determined that in the worst case $|g(t)|(\Delta t)^2 \leq .01$ in the interval under consideration. Since $a(t)$ is computed first, all the $g(t)$ are known, and therefore the successive intervals can be determined beforehand. They are then used in the computation of the integrals, as mentioned above. Since all the $g(t)$ vary rapidly near the minima of $a(t)$, the intervals Δt change widely. We found that $\Delta t \sim .0005$ near the minima of $a(t)$ and it increases to $.1715$ near the maxima of $a(t)$.

$\lambda(\theta,t)$ and $b(t)$ are obtained, according to formulae (4.5,6) by multiplying the bubble coefficients by appropriate factors depending on σ, μ, and ν and adding.

VI. Discussion of Results

We assume that the bubble motion is adequately described by considering only terms up to third order in σ. Because of the structure of the series (4.6), the equation for the cross section of the bubble is then of the form

$$\lambda(\theta,t) = A + B\cos\theta + C\cos^2\theta + D\cos^3\theta \; . \tag{6.1}$$

This, of course, severely limits the number of possible shapes that the bubble can take on according to the present theory.

Case 1: Bubble produced by exploding 300 lb. TNT at 158' head in infinite water.

Figure 1 shows the computed migration of the bubble up to $t = 2.977$ at which time the center of the bubble reaches its surface and the representation for the potential, equation (4.4), becomes meaningless. For comparison, we traced the migration predicted by the first order theory. At time of breakdown, the former predicts $b = .50$ while the latter, $b = .87$, or 74°/o more. Notice that after $t = 2.95$ the third order theory predicts a downward migration, which is rather unexpected and indicates that these results cannot be trusted beyond that time. At $t = 2.95$, $b = .53$ according to the third order theory, while $b = .77$ according to the first order theory, or 45.5°/o more. This is in excellent agreement with experiments.[2]

[2] According to Dr. Arons, the Herring rise formula predicts for explosions of moderate size, a migration at time of the first secondary pulse about 50°/o in excess of the observed migration. Oral communication.

The bubble takes on a shape in accordance with the qualitative description in [3]. Near the time of the secondary pulse it first flattens and then becomes kidney shaped. A picture of the moving bubble is shown in Figure 2. We note that at the time of breakdown, the compression factor is 9.5 while the compression factor for the spherical bubble is (for $x = .1957$) about 575.

Case 2: Bubble produced by exploding 300 lb. TNT at 158' head in water 145' deep (bottom 20' below the explosion center).

It is expected that in this case the attraction of the bottom will make the bubble appear almost stationary. This is exactly what our calculations predict, (see Figure 3). At the end of the period the bottom of the bubble seems even to move downward. It is then that the upward pull due to gravity and downward pull due to the proximity of the bottom result in an elongation of the bubble in the vertical direction. As shown in Fig. 4, these opposing forces will eventually result in a splitting of the bubble into two smaller bubbles.

Case 3: Bubble produced by exploding 1.05 g TNT at 158' head in infinite water.

Fig. 5 shows the migration of the center of gravity for two periods. During the first period the bubble remains practically spherical. At the early stages of second expansion there is a flattening of the lower part of the bubble surface. See stages 2 and 3, Fig. 6 However, at the time of the full expansion the bubble is again almost spherical. Subsequent motion is similar to that described in Case 1, and the motion can be tracked up to $t = 6.200$.

The first bubble maximum and period, as given by the present theory, are roughly the same as those of the lowest approximation, which are tabulated in [2].

Table 1

x	f(x)	f'(x)	f"(x)
0	0	1.832	0
.05	.092	1.847	.602
.10	.185	1.892	1.268
.15	.282	1.970	1.961
.20	.383	2.09	2.72
.25	.491	2.25	3.65
.30	.608	2.46	4.90
.35	.378	2.74	6.52
.40	.884	3.11	8.47
.45	1.050	3.60	11.23
.50	1.246	4.26	15.45
.55	1.481	5.17	21.5
.60	1.769	6.45	30.8
.65	2.14	8.33	46.8
.70	2.62	11.25	75.5
.75	3.29	16.12	130.5
.80	4.30	25.1	249
.85	5.97	44.5	630
.90	9.30	100.0	2,560
.95	19.3	400	31,000
1.00	∞	∞	∞

log 2 = .692 $\qquad \frac{3}{16}\sum_{s=1}^{\infty} s^{-3} = .226$

Table 2

Bubble coefficients for $\varkappa = .1957$

t	a	$\dot{a}$	$\ddot{a}$
0	.19	0	864.00
.0005	.19	.45	859.40
.0015	.20	1.25	832.10
.0025	.20	2.10	776.00
.0035	.20	2.79	706.80
.0050	.20	3.80	571.20
.0065	.21	4.54	441.30
.0085	.22	5.26	285.60
.0105	.23	5.70	165.68
.0135	.24	6.00	46.34
.0185	.28	5.98	-40.09
.0255	.32	5.57	-66.85
.0325	.36	5.10	-63.90
.0405	.40	4.64	-54.24
.0495	.44	4.19	-43.96
.0595	.48	3.80	-35.31
.0695	.51	3.48	-28.69
.0795	.55	3.22	-23.91
.0895	.58	3.00	-20.17
.1045	.62	2.73	-16.13
.1195	.66	2.51	-13.25
.1395	.71	2.28	-10.53
.1595	.75	2.09	- 8.62
.1795	.79	1.93	- 7.22
.2095	.85	1.73	- 5.72
.2395	.90	1.58	- 4.70
.2795	.96	1.41	- 3.74
.3295	1.02	1.25	- 2.94
.3795	1.08	1.12	- 2.40
.4395	1.14	.99	- 1.96
.5095	1.21	.86	- 1.60
.5895	1.27	.75	- 1.33
.6795	1.33	.64	- 1.11
.7795	1.39	.53	- .95
.8795	1.44	.45	- .84
.9795	1.48	.37	- .75
1.0795	1.52	.29	- .69
1.1795	1.54	.23	- .65
1.2795	1.56	.16	- .62
1.3795	1.57	.10	- .60
1.5510	1.58	0	- .59

a, $\dot{a}$, and $\ddot{a}$ are periodic with period 3.1020.

a and $\ddot{a}$ are even functions, $\dot{a}$ is odd.

Table 3

Bubble coefficients for $\varkappa = .1957$

Numbers in parentheses are exponents of 10 .

t	b_1	$\dot{b}_1$	a_{10}	β_2'
.1195		.100 (0)		
.1395		.118		
.1595		.135		
.1795		.153		
.2095		.179	- .106	
.2395		.206	- .118	
.2795		.243	- .131	
.3295		.289	- .147	
.3795		.337	- .160	
.4395		.395	- .175	
.5095	.115(0)	.464	- .188	
.5895	.155	.546	- .200	
.6795	.208	.641	- .208	
.7795	.278	.752	- .211	- 1.06
.8795	.359	.868	- .208	- 1.12
.9795	.452	.991	- .196	- 1.15
1.0795	.558	1.12	- .180	- 1.14
1.1795	.677	1.26	- .156	- 1.10
1.2795	.810	1.41	- .125	- 1.03
1.3795	.959	1.57	- .0865	- .914
1.5510	1.26	1.89	0	- .622

Table 3 (continued)

t	b_1	$\dot{b}_1$	a_{10}	β_2'
1.7225	1.61	2.26	.114	- .200
1.8225	1.85	2.52	.195	.111
1.9225	2.12	2.82	.288	.474
2.0225	2.42	3.18	.396	.894
2.1225	2.76	3.60	.522	1.38
2.2225	3.14	4.12	.667	1.93
2.3225	3.59	4.78	.838	2.56
2.4225	4.11	5.65	1.04	3.30
2.5125	4.67	6.72	1.26	4.06
2.5925	5.26	8.02	1.50	4.86
2.6625	5.87	9.61	1.76	5.67
2.7225	6.51	1.15 (1)	2.03	6.48
2.7725	7.14	1.37	2.30	7.27
2.8225	7.91	1.69	2.64	8.20
2.8625	8.66	2.06	2.99	9.10
2.8925	9.34	2.46	3.30	9.90
2.9225	1.02 (1)	3.01	3.69	1.08 (1)
2.9425	1.08	3.52	4.01	1.16
2.9625	1.15	4.22	4.39	1.25
2.9825	1.25	5.21	4.86	1.36
2.9975	1.34	6.26	5.30	1.45
3.0125	1.44	7.77	5.84	1.57
3.0225	1.53	9.18	6.28	1.66
3.0325	1.63	1.11 (2)	6.79	1.78
3.0425	1.76	1.39	7.43	1.91
3.0525	1.91	1.80	8.25	2.09
3.0615	2.10	2.40	9.07	2.30
3.0695	2.33	3.27	9.98	2.55
3.0765	2.60	4.55	1.09 (1)	2.85
3.0835	3.00	6.82	1.17	3.28
3.0885	3.41	9.62	1.18	3.73
3.0915	3.74	1.20 (3)	1.12	4.09
3.0935	4.00	1.39	1.03	4.37
3.0955	4.29	1.59	8.91 (0)	4.70
3.0970	4.54	1.74	7.46	4.97
3.0985	4.82	1.88	5.48	5.27
3.0995	5.01	1.94	4.11	5.48
3.1005	5.20	2.00	2.45	5.69
3.1015	5.40	2.02	.886	5.91
3.1020	5.50	2.03	0	6.02

Table 3 (continued)

t	b_1	$\dot{b}_1$	α_{10}	β_2'
3.1025	5.61 (1)	2.02 (3)	- .886 (0)	6.14 (1)
3.1035	5.81	2.00	-2.45	6.36
3.1045	6.00	1.94	-4.11	6.57
3.1055	6.20	1.88	-5.48	6.78
3.1070	6.47	1.74	-7.46	7.08
3.1085	6.72	1.59	-8.91	7.35
3.1105	7.01	1.39	-1.03 (1)	7.68
3.1125	7.27	1.20	-1.12	7.96
3.1155	7.60	9.62 (2)	-1.18	8.31
3.1205	8.01	6.82	-1.17	8.76
3.1275	8.41	4.55	-1.10	9.20
3.1345	8.68	3.27	-1.00	9.50
3.1425	8.91	2.40	-9.13 (0)	9.74
3.1515	9.10	1.80	-8.26	9.95
3.1615	9.26	1.39	-7.50	1.01 (2)
3.1715	9.38	1.11	-6.88	1.02
3.1815	9.48	9.20 (1)	-6.37	1.04
3.1915	9.57	7.78	-5.94	1.05
3.2065	9.67	6.28	-5.42	1.06
3.2215	9.76	5.23	-5.00	1.07
3.2415	9.85	4.25	-4.55	1.08
3.2615	9.93	3.55	-4.18	1.08
3.2815	1.00 (2)	3.04	-3.88	1.09
3.3115	1.01	2.49	-3.52	1.10
3.3415	1.01	2.10	-3.22	1.11
3.3815	1.02	1.74	-2.91	1.11
3.4315	1.03	1.43	-2.60	1.12
3.4815	1.04	1.22	-2.35	1.13
3.5415	1.04	1.04	-2.11	1.13
3.6115	1.05	8.94 (0)	-1.88	1.14
3.6915	1.06	7.81	-1.66	1.15
3.7815	1.06	6.94	-1.46	1.15
3.8815	1.07	6.29	-1.26	1.16
3.9815	1.08	5.86	-1.08	1.16
4.0815	1.08	5.59	- .916	1.17
4.1815	1.09	5.42	- .757	1.17
4.2815	1.09	5.35	- .601	1.18
4.3815	1.10	5.34	- .445	1.18
4.4815	1.10	5.41	- .287	1.19
4.6530	1.11	5.66	0	1.20

Table 3 (continued)

t	b_1	$\dot{b}_1$	α_{10}	β_2'
4.8245	1.12	6.10	.315	1.21
4.9245	1.13	6.46	.515	1.22
5.0245	1.14	6.91	.733	1.22
5.1245	1.14	7.48	.973	1.23
5.2245	1.15	8.20	1.24	1.24
5.3245	1.16	9.12	1.54	1.25
5.4245	1.17	1.03 (1)	1.89	1.27
5.5245	1.18	1.19	2.29	1.28
5.6145	1.19	1.40	2.73	1.30
5.6945	1.21	1.65	3.20	1.31
5.7645	1.22	1.96	3.70	1.32
5.8245	1.23	2.34	4.22	1.34
5.8745	1.24	2.78	4.76	1.36
5.9245	1.26	3.41	5.42	1.37
5.9645	1.27	4.15	6.09	1.39
5.9945	1.29	4.93	6.71	1.40
6.0245	1.30	6.04	7.48	1.42
6.0445	1.32	7.06	8.11	1.44
6.0645	1.33	8.46	8.86	1.46
6.0845	1.35	1.04 (2)	9.80	1.48
6.0995	1.37	1.25	1.07 (1)	1.50
6.1145	1.39	1.55	1.17	1.52
6.1245	1.41	1.84	1.26	1.54
6.1345	1.43	2.22	1.36	1.56
6.1445	1.45	2.77	1.49	1.59
6.1545	1.48	3.60	1.64	1.62
6.1635	1.52	4.80	1.82	1.66
6.1715	1.57	6.55	2.00	1.71
6.1785	1.62	9.10	2.19	1.77
6.1855	1.70	1.36 (3)	2.34	1.86
6.1905	1.78	1.92	2.36	1.95
6.1935	1.85	2.40	2.24	2.02
6.1955	1.90	2.78	2.06	2.08
6.1975	1.96	3.18	1.78	2.14
6.1990	2.01	3.48	1.49	2.20
6.2005	2.06	3.76	1.10	2.26
6.2015	2.10	3.89	8.22 (0)	2.30
6.2025	2.14	4.00	4.91	2.34
6.2035	2.18	4.05	4.77	2.39
6.2040	2.20	4.06	0	2.41

Tablo 4

t	$\tilde{b}_3$	$\tilde{a}_{20}$	a_{22}	a_{33}
1.7225	0	0	- 1.02	0
1.8225			- 1.35	
1.9225			- 1.78	
2.0225			- 2.34	
2.1225			- 3.08	
2.2225		1.20	- 4.07	1.19 (1)
2.3225	- 1.08 (1)	1.55	- 5.40	1.83
2.4225	- 1.70	2.03	- 7.25	2.87
2.5125	- 2.61	2.62	- 9.60	4.41
2.5925	- 3.95	3.34	- 1.25(1)	6.63
2.6625	- 5.86	4.19	- 1.61	9.74
2.7225	- 8.50	5.18	- 2.03	1.40 (2)
2.7725	- 1.20 (2)	6.28	- 2.52	1.95
2.8225	- 1.76	7.80	- 3.19	2.82
2.8625	- 2.49	9.47	- 3.95	3.95
2.8925	- 3.34	1.12 (1)	- 4.72	5.23
2.9225	- 4.67	1.34	- 5.78	7.21
2.9425	- 5.99	1.54	- 6.73	9.17
2.9625	- 7.94	1.80	- 7.97	1.20 (3)
2.9825	- 1.10 (3)	2.14	- 9.66	1.64
2.9975	- 1.45	2.49	- 1.14(2)	2.14
3.0125	- 1.20	2.96	- 1.38	2.92
3.0225	- 2.55	3.37	- 1.60	3.72
3.0325	- 3.37	3.91	- 1.89	4.88
3.0425	- 4.67	4.64	- 2.29	6.72
3.0525	- 6.89	5.66	- 2.88	9.86
3.0615	- 1.06 (4)	7.04	- 3.73	1.52 (4)
3.0695	- 1.70	8.90	- 4.96	2.46
3.0765	- 2.85	1.14 (2)	- 6.82	4.23
3.0835	- 5.62	1.55	- 1.04(3)	8.74
3.0885	- 1.03 (5)	2.02	- 1.52	1.70 (5)
3.0915	- 1.56	2.41	- 2.02	2.73
3.0935	- 2.11	2.71	- 2.48	3.88
3.0955	- 2.94	3.04	- 3.11	5.65
3.0970	- 3.79	3.30	- 3.72	7.59
3.0985	- 4.93	3.55	- 4.49	1.03 (6)
3.0995	- 5.86	3.70	- 5.08	1.27
3.1005	- 6.94	3.83	- 5.77	1.56
3.1015	- 8.18	3.94	- 6.53	1.92
3.1020	- 8.87	3.98	- 6.94	2.12

Bubble Coefficients for $\varkappa$ = .1957

Numbers in parentheses are exponents of 10 .

Table 4 (continued)

t	$\tilde{b}_3$	$\tilde{a}_{20}$	a_{22}	a_{33}
3.1025	- 9.60 (5)	4.02 (2)	- 7.37 (3)	2.34 (6)
3.1035	- 1.12 (6)	4.07	- 8.29	2.84
3.1045	- 1.29	4.07	- 9.25	3.40
3.1055	- 1.47	4.06	- 1.03 (4)	4.04
3.1070	- 1.75	3.98	- 1.18	5.08
3.1085	- 2.04	3.90	- 1.34	6.20
3.1105	- 2.41	3.69	- 1.54	7.77
3.1125	- 2.76	3.46	- 1.72	9.31
3.1155	- 3.21	3.12	- 1.96	1.15 (7)
3.1205	- 3.82	2.64	- 2.26	1.44
3.1275	- 4.40	2.14	- 2.55	1.73
3.1345	- 4.79	1.80	- 2.73	1.92
3.1425	- 5.10	1.52	- 2.86	2.04
3.1515	- 5.34	1.30	- 2.94	2.11
3.1615	- 5.53	1.12	- 2.98	2.13
3.1715	- 5.66	9.90 (1)	- 2.98	2.10
3.1815	- 5.77	8.87	- 2.96	2.06
3.1915	- 5.85	8.03	- 2.94	2.00
3.2065	- 5.94	7.06	- 2.88	1.91
3.2215	- 6.01	6.30	- 2.82	1.80
3.2415	- 6.08	5.52	- 2.72	1.66
3.2615	- 6.14	4.91	- 2.63	1.52
3.2815	- 6.17	4.43	- 2.53	1.39
3.3115	- 6.22	3.86	- 2.40	1.21
3.3415	- 6.25	3.42	- 2.26	1.04
3.3815	- 6.28	2.98	- 2.10	8.41 (6)
3.4315	- 6.32	2.58	- 1.91	6.22
3.4815	- 6.34	2.29	- 1.73	4.31
3.5415	- 6.35	2.04	- 1.54	2.31
3.6115	- 6.37	1.86	- 1.35	2.91 (5)
3.6915	- 6.38	1.74	- 1.14	- 1.68 (6)
3.7815	- 6.39	1.71	- 9.40 (3)	- 3.57
3.8815	- 6.40	1.76	- 7.36	- 5.34
3.9815	- 6.40	1.90	- 5.49	- 6.85
4.0815	- 6.41	2.11	- 3.75	- 8.14
4.1815	- 6.41	2.39	- 2.12	- 9.26
4.2815	- 6.41	2.73	- 5.76 (2)	- 1.02 (7)
4.3815	- 6.41	3.14	- 9.05	- 1.10
4.4815	- 6.41	3.62	2.34 (3)	- 1.18
4.6530	- 6.41	4.63	4.71	- 1.28

Table 4 (continued)

t	$\tilde{b}_3$	$\tilde{a}_{20}$	a_{22}	a_{33}
4.8245	- 6.40	5.89	7.03	- 1.35
4.9245	- 6.39	6.77	8.37	- 1.38
5.0245	- 6.39	7.78	9.72	- 1.39
5.1245	- 6.38	8.95	1.11 (4)	- 1.40
5.2245	- 6.37	1.03 (2)	1.25	- 1.40
5.3245	- 6.35	1.19	1.39	- 1.37
5.4245	- 6.33	1.38	1.54	- 1.33
5.5245	- 6.31	1.61	1.69	- 1.27
5.6145	- 6.28	1.87	1.82	- 1.19
5.6945	- 6.25	2.15	1.95	- 1.08
5.7645	- 6.21	2.46	2.06	- 9.60 (6)
5.8245	- 6.16	2.80	2.17	- 8.21
5.8745	- 6.12	3.15	2.25	- 6.74
5.9245	- 6.05	3.57	2.33	- 4.87
5.9645	- 5.99	4.05	2.39	- 2.97
5.9945	- 5.92	4.48	2.44	- 1.22
6.0245	- 5.83	5.02	2.47	8.93 (5)
6.0445	- 5.76	5.46	2.48	2.57 (6)
6.0645	- 5.67	6.01	2.48	4.53
6.0845	- 5.55	6.70	2.46	6.84
6.0995	- 5.44	7.35	2.44	8.86
6.1145	- 5.31	8.17	2.38	1.12 (7)
6.1245	- 5.19	8.85	2.32	1.30
6.1345	- 5.05	9.68	2.24	1.50
6.1445	- 4.86	1.07 (3)	2.13	1.73
6.1545	- 4.63	1.20	1.96	1.99
6.1635	- 4.36	1.36	1.74	2.26
6.1715	- 4.08	1.54	1.43	2.57
6.1785	- 3.80	1.75	1.04	2.93
6.1855	- 3.40	2.00	4.20 (3)	3.45
6.1905	- 3.43	2.19	- 2.80	4.07
6.1935	- 3.92	2.28	- 8.79	4.65
6.1955	- 4.55	2.29	- 1.39 (4)	5.20
6.1975	- 5.60	2.25	- 2.01	5.96
6.1990	- 6.75	2.18	- 2.57	6.71
6.2005	- 8.33	2.06	- 3.22	7.71
6.2015	- 9.64	1.94	- 3.71	8.52
6.2025	- 1.12 (7)	1.81	- 4.24	9.50
6.2035	- 1.30	1.66	- 4.82	1.06 (8)
6.2040	- 1.49	1.57	- 5.13	1.12

Table 5

t	a_{20}	a_{30}	a'_{30}	a''_{30}	a_{32}
1.7225	0	0	0	0	0
1.8225					
1.9225					
2.0225	- 1.07				
2.1225	- 1.34				
2.2225	- 1.65				
2.3225	- 2.05				
2.4225	- 2.55				1.53 (1)
2.5125	- 3.16				2.54
2.5925	- 3.88	- 1.18 (1)			4.11
2.6625	- 4.75	- 1.65		1.25 (1)	6.46
2.7225	- 5.78	- 2.29		1.68	9.88
2.7725	- 6.98	- 3.12		2.26	1.46 (2)
2.8225	- 8.71	- 4.46		3.24	2.26
2.8625	- 1.07(1)	- 6.20	1.07 (1)	4.58	3.36
2.8925	- 1.29	- 8.25	1.37	6.20	4.69
2.9225	- 1.59	- 1.14 (2)	1.78	9.68	6.[illegible]4
2.9425	- 1.88	- 1.47	2.15	1.17 (2)	9.09
2.9625	- 2.26	- 1.94	2.63	1.60	1.25 (3)
2.9825	- 2.81	- 2.68	3.29	2.28	1.80
2.9975	- 3.38	- 3.55	3.95	3.10	2.48
3.0125	- 4.18	- 4.90	4.86	4.37	3.56
3.0225	- 4.92	- 6.25	5.68	5.68	4.72
3.0325	- 5.86	- 8.23	6.74	7.45	6.47
3.0425	- 7.16	- 1.13 (3)	8.19	1.00 (3)	9.36
3.0525	- 8.86	- 1.62	1.03 (2)	1.34	1.45 (4)
3.0615	- 1.09(2)	- 2.39	1.31	1.64	2.35
3.0695	- 1.28	- 3.56	1.70	1.42	3.98
3.0765	- 1.34	- 5.28	2.23	- 7.54 (2)	7.07
3.0835	- 8.27(1)	- 7.94	3.11	- 1.13 (4)	1.55 (5)
3.0885	8.36	- 9.82	4.14	- 3.60	2.85
3.0915	3.14(2)	- 9.44	5.00	- 6.49	4.30
3.0935	5.45	- 7.20	5.69	- 8.84	5.74
3.0955	8.45	- 1.97	6.44	- 1.09 (5)	7.70
3.0970	1.10(3)	4.74	7.04	- 1.14	9.56
3.0985	1.36	1.43 (4)	7.64	- 1.02	1.18 (6)
3.0995	1.49	2.20	8.00	- 8.37 (4)	1.33
3.1005	1.60	3.06	8.33	- 5.38	1.50
3.1015	1.66	3.94	8.60	- 2.09	1.66
3.1020	1.66	4.39	8.72	- 1.45 (3)	1.73

Bubble Coefficients for $\varkappa$ = .1957

Numbers in parentheses are exponents of 10.

Table 5 (continued)

t	a_{20}	a_{30}	a'_{30}	a''_{30}	a_{32}
3.1025	1.66 (3)	4.82 (4)	8.82([illegible])	1.80 (4)	1.81 (6)
3.1035	1.60	5.57	8.96	5.10	1.94
3.1045	1.50	6.14	9.02	8.11	2.04
3.1055	1.36	6.52	9.02	9,96	2.12
3.1070	1.10	6.67	8.88	1.12 (5)	2.17
3.1085	8.54 (2)	6.36	8.65	1.07	2.15
3.1105	5.55	5.44	8.24	8.76 (4)	2.05
3.1125	3.24	4.13	7.77	6.44	1.88
3.1155	9.48 (1)	1.98	7.04	3.59	1.58
3.1205	- 7.17	- 1.23	6.00	1.14	1.10
3.1275	- 1.24 (2)	- 2.50	4.90	9.59 (2)	5.99 (5)
3.1345	- 1.18	- 4.96	4.14	- 1.24 (3)	2.72
3.1425	- 1.00	- 6.90	3.52	- 1.48	2.90
3.1515	- 8.07 (1)	- 8.42	3.01	- 1.22	- 1.49 (5)
3.1615	- 6.44	- 9.66	2.60	- 9.05 (2)	- 2.77
3.1715	- 5.19	- 1.06 (5)	2.28	- 6.63	- 3.64
3.1815	- 4.29	- 1.13	2.04	- 4.99	- 4.25
3.1915	- 3.58	- 1.19	1.84	- 3.80	- 4.70
3.2065	- 2.83	- 1.27	1.60	- 2.64	- 5.17
3.2215	- 2.29	- 1.33	1.42	- 1.89	- 5.49
3.2415	- 1.78	- 1.40	1.22	- 1.28	- 5.77
3.2615	- 1.43	- 1.46	1.06	- 9.08 (1)	- 5.94
3.2815	- 1.18	- 1.52	9.30(1)	- 6.66	- 6.05
3.3115	- 9.07 (0)	- 1.58	7.72	- 4.40	- 6.13
3.3415	- 7.23	- 1.65	6.44	- 3.06	- 6.16
3.3815	- 5.54	- 1.72	5.08	- 1.99	- 6.14
3.4315	- 4.18	- 1.80	3.73	- 1.23	- 6.09
3.4815	- 3.28	- 1.88	2.67	- 8.03 (0)	- 5.95
3.5415	- 2.58	- 1.96	1.66	- 4.98	- 5.80
3.6115	- 2.06	- 2.06	7.48(0)	- 2.83	- 5.60
3.6915	- 1.69	- 2.16	- 1.25(-1)	- 1.35	- 5.36
3.7815	- 1.46	- 2.26	- 5.88(0)	- 2.67 (-1)	- 5.10
3.8815	- 1.35	- 2.38	- 9.38	5.79	- 4.80
3.9815	- 1.33	- 2.49	- 1.01(1)	1.25 (0)	- 4.51
4.0815	- 1.38	- 2.60	- 8.38(0)	1.85	- 4.22
4.1815	- 1.48	- 2.72	- 4.17	2.43	- 3.94
4.2815	- 1.63	- 2.83	2.47	3.03	- 3.67
4.3815	- 1.82	- 2.95	1.15(1)	3.67	- 3.39
4.4815	- 2.06	- 3.08	2.32	4.37	- 3.12
4.6530	- 2.46	- 3.30	4.95	5.78	- 2.64

Table 5 (continued)

t	a_{20}	a_{30}	a'_{30}	a''_{30}	a_{32}
4.8245	- 3.21	- 3.56	8.49	7.59	- 2.14
4.9245	- 3.68	- 3.72	1.10 (2)	8.91	- 1.84
5.0245	- 4.23	- 3.91	1.40	1.05 (1)	- 1.53
5.1245	- 4.88	- 4.11	1.75	1.25	- 1.20
5.2245	- 5.67	- 4.34	2.15	1.52	- 8.46 (4)
5.3245	- 6.65	- 4.61	2.62	1.87	- 4.69
5.4245	- 7.88	- 4.91	3.19	2.36	- 5.95 (3)
5.5245	- 9.53	- 5.29	3.88	3.11	3.94 (4)
5.6145	- 1.15 (1)	- 5.70	4.63	4.17	8.54
5.6945	- 1.40	- 6.15	5.46	5.70	1.32 (5)
5.7645	- 1.72	- 6.65	6.36	7.92	1.78
5.8245	- 2.09	- 7.19	7.32	1.10 (2)	2.24
5.8745	- 2.54	- 7.74	8.30	1.53	2.68
5.9245	- 3.19	- 8.45	9.54	2.38	3.19
5.9645	- 3.96	- 9.18	1.08 (3)	3.27	3.70
5.9945	- 4.78	- 9.86	1.20	4.50	4.15
6.0245	- 5.97	- 1.07 (6)	1.35	6.55	4.71
6.0445	- 7.08	- 1.14	1.47	8.72	5.17
6.0645	- 8.59	- 1.23	1.62	1.20 (3)	5.74
6.0845	- 1.07 (2)	- 1.34	1.80	1.74	6.49
6.0995	- 1.30	- 1.44	1.98	2.37	7.25
6.1145	- 1.61	- 1.56	2.19	3.37	8.28
6.1245	- 1.90	- 1.66	2.38	4.39	9.23
6.1345	- 2.28	- 1.78	2.59	5.78	1.05 (6)
6.1445	- 2.79	- 1.93	2.86	7.82	1.23
6.1545	- 3.46	- 2.12	3.21	1.05 (4)	1.50
6.1635	- 4.26	- 2.32	3.62	1.28	1.90
6.1715	- 5.02	- 2.54	4.08	1.10	2.50
6.1785	- 5.25	- 2.74	4.59	6.46 (3)	3.40
6.1855	- 3.20	- 2.87	5.19	9.04	5.12
6.1905	3.46	- 2.77	5.61	2.88 (4)	7.65
6.1935	1.26 (3)	- 2.49	5.74	5.18 (5)	9.88
6.1955	2.19	- 2.15	5.71	7.05	1.19 (7)
6.1975	3.39	- 1.62	5.53	8.67	1.44
6.1990	4.39	- 1.10	5.26	9.09	1.66
6.2005	5.44	- 4.65 (5)	4.86	8.10	1.91
6.2015	5.97	1.63 (3)	4.51	6.64	2.09
6.2025	6.41	4.86 (5)	4.11	4.25	2.26
6.2035	6.62	9.66	3.66	1.62	2.43
6.2040	6.66	1.21 (6)	3.42	5.80 (3)	2.50

Table 6

t	α'_{32}	α''_{32}	β_2	β_3	β'_3
1.7225	0	0	0	0	0
1.8225					
1.9225					
2.0225					
2.1225			- 1.18 (0)		
2.2225			- 1.67		
2.3225	- 1.12 (1)		- 2.37		
2.4225	- 1.58		- 3.41	1.02 (1)	- 1.21 (1)
2.5125	- 2.11		- 4.81	1.59	- 1.48
2.5925	- 2.79	1.13 (1)	- 6.67	2.46	- 1.79
2.6625	- 3.63	1.27	- 9.09	3.75	- 2.16
2.7225	- 4.63	1.39	- 1.22 (1)	5.63	- 2.59
2.7725	- 5.77	1.50	- 1.58	8.24	- 3.09
2.8225	- 7.35	1.61	- 2.14	1.28 (2)	- 3.80
2.8625	- 9.12	1.70	- 2.80	1.91	- 4.62
2.8925	- 1.09 (2)	1.77	- 3.51	2.70	- 5.49
2.9225	- 1.33	1.84	- 4.54	4.01	- 6.71
2.9425	- 1.55	1.89	- 5.50	5.40	- 7.83
2.9625	- 1.83	1.93	- 6.82	7.59	- 9.36
2.9825	- 2.22	1.96	- 8.72	1.12 (3)	- 1.15 (2)
2.9975	- 2.61	1.99	- 1.08 (2)	1.58	- 1.38
3.0125	- 3.16	2.00	- 1.37	2.33	- 1.72
3.0225	- 3.64	2.00	- 1.64	3.13	- 2.02
3.0325	- 4.29	2.00	- 2.02	4.38	- 2.44
3.0425	- 5.19	1.96	- 2.56	6.47	- 3.05
3.0525	- 6.51	1.91	- 3.39	1.02 (4)	- 3.97
3.0615	- 8.38	1.84	- 4.59	1.67	- 5.29
3.0695	- 1.11 (3)	1.72	- 6.35	2.81	- 7.23
3.0765	- 1.52	1.56	- 8.95	4.79	- 1.01 (3)
3.0835	- 2.30	1.32	- 1.36 (3)	8.83	- 1.52
3.0885	- 3.39	1.07	- 1.91	1.37 (5)	- 2.12
3.0915	- 4.47	8.57 (0)	- 2.37	1.72	- 2.64
3.0935	- 5.49	6.89	- 2.73	1.91	- 3.04
3.0955	- 6.88	4.97	- 3.12	1.96	- 3.47
3.0970	- 8.20	3.37	- 3.40	1.82	- 3.80
3.0985	- 9.80	1.66	- 3.66	1.49	- 4.08
3.0995	- 1.12 (4)	4.50 (-1)	- 3.79	1.14	- 4.23
3.1005	- 1.27	- 7.92 (-1)	- 3.88	7.08 (4)	- 4.34
3.1015	- 1.43	- 2.06 (0)	- 3.93	2.14	- 4.40
3.1020	- 1.52	- 2.70	- 3.93	- 4.42 (3)	- 4.41

Bubble Coefficients for $x = .1957$

Numbers in parentheses are exponents of 10 .

Table 6 (continued)

t	α'_{32}	α''_{32}	β_2	β_3	β'_3
3.1025	- 1.62 (4)	-3.35 (0)	- 3.92 (3)	- 3.02 (4)	- 4.40 (3)
3.1035	- 1.82	-4.64	- 3.87	- 7.94	- 4.34
3.1045	- 2.03	-5.92	- 3.76	- 1.22 (5)	- 4.24
3.1055	- 2.26	-7.19	- 3.62	- 1.56	- 4.09
3.1070	- 2.60	-9.01	- 3.35	- 1.89	- 3.81
3.1085	- 2.94	-1.07 (1)	- 3.05	- 2.01	- 3.49
3.1105	- 3.38	-1.28	- 2.65	- 1.94	- 3.06
3.1125	- 3.78	-1.47	- 2.27	- 1.73	- 2.65
3.1155	- 4.30	-1.70	- 1.79	- 1.36	- 2.15
3.1205	- 4.96	-1.98	- 1.22	- 8.49 (4)	- 1.54
3.1275	- 5.60	-2.22	- 7.33 (2)	- 4.23	- 1.04
3.1345	- 6.00	-2.38	- 4.57	- 2.12	- 7.53 (2)
3.1425	- 6.28	-2.48	- 2.67	- 8.91 (3)	- 5.61
3.1515	- 6.45	-2.54	- 1.36	- 1.80 (3)	- 4.30
3.1615	- 6.54	-2.56	- 4.39 (1)	2.40	- 3.39
3.1715	- 6.56	-2.56	1.77 (1)	4.77	- 2.80
3.1815	- 6.55	-2.54	6.14	6.22	- 2.38
3.1915	- 6.50	-2.51	9.38	7.17	- 2.08
3.2065	- 6.40	-2.46	1.30 (2)	8.08	- 1.75
3.2215	- 6.27	-2.40	1.55	8.65	- 1.52
3.2415	- 6.08	-2.15	1.80	9.12	- 1.31
3.2615	- 5.88	-2.24	1.98	9.41	- 1.16
3.2815	- 5.68	-2.16	2.11	9.60	- 1.05
3.3115	- 5.39	-2.04	2.26	9.80	- 9.30 (1)
3.3415	- 5.10	-1.93	2.38	9.92	- 8.46
3.3815	- 4.75	-1.80	2.49	1.00 (4)	- 7.67
3.4315	- 4.33	-1.64	2.59	1.01	- 7.00
3.4815	- 3.92	-1.51	2.67	1.02	- 6.59
3.5415	- 3.51	-1.36	2.74	1.02	- 6.16
3.6115	- 3.08	-1.21	2.80	1.02	- 5.85
3.6915	- 2.64	-1.06	2.86	1.02	- 5.67
3.7815	- 2.19	-9.05 (0)	2.91	1.03	- 5.45
3.8815	- 1.74	-7.51	2.95	1.03	- 5.35
3.9815	- 1.33	-6.09	2.99	1.03	- 5.30
4.0815	- 9.48 (3)	-4.71	3.02	1.03	- 5.24
4.1815	- 5.88	-3.35	3.04	1.03	- 5.32
4.2815	- 2.46	-1.97	3.07	1.03	- 5.38
4.3815	8.24 (2)	-5.44 (-1)	3.09	1.03	- 5.46
4.4815	4.00 (3)	9.39 (-1)	3.10	1.03	- 5.57
4.6530	9.28	3.67 (0)	3.13	1.03	- 5.82

Table 6 (continued)

t	α'_{32}	α''_{32}	β_2	β_3	β'_3
4.8245	1.44 (4)	6.70	3.15	1.03	- 6.15
4.9245	1.74	8.62	3.16	1.03	- 6.39
5.0245	2.05	1.07 (1)	3.16	1.03	- 6.67
5.1245	2.36	1.29	3.17	1.03	- 6.99
5.2245	2.67	1.52	3.17	1.03	- 7.38
5.3245	2.99	1.77	3.17	1.04	- 7.86
5.4245	3.31	2.04	3.15	1.04	- 8.46
5.5245	3.60	2.32	3.12	1.04	- 9.24
5.6145	3.97	2.59	3.08	1.04	- 1.02 (2)
5.6945	4.26	2.84	3.02	1.05	- 1.13
5.7645	4.52	3.06	2.93	1.06	- 1.27
5.8245	4.74	3.26	2.82	1.07	- 1.44
5.8745	4.93	3.42	2.69	1.09	- 1.63
5.9245	5.12	3.59	2.48	1.12	- 1.91
5.9645	5.26	3.73	2.23	1.17	- 2.24
5.9945	5.35	3.82	1.96	1.23	- 2.58
6.0245	5.43	3.91	1.57	1.33	- 3.06
6.0445	5.46	3.95	1.20	1.43	- 3.51
6.0645	5.47	3.99	6.84 (1)	1.60	- 4.12
6.0845	5.44	4.00	- 5.74 (0)	1.88	- 4.99
6.0995	5.38	4.00	- 8.54 (1)	2.23	- 5.91
6.1145	5.28	3.94	- 2.01 (2)	2.82	- 7.23
6.1245	5.16	3.89	- 3.08	3.45	- 8.46
6.1345	4.99	3.81	- 4.57	4.43	- 1.01 (3)
6.1445	4.75	3.68	- 6.71	6.06	- 1.25
6.1545	4.38	3.49	- 1.00 (3)	9.03	- 1.62
6.1635	3.88	3.23	- 1.48	1.42 (5)	- 2.15
6.1715	3.22	2.88	- 2.18	2.32	- 2.92
6.1785	2.36	2.42	- 3.21	3.88	- 4.06
6.1855	1.02	1.74	- 5.05	7.08	- 6.09
6.1905	- 5.06 (3)	1.03	- 7.25	1.09 (6)	- 8.52
6.1935	- 1.81 (4)	4.53 (0)	- 9.08	1.37	- 1.06 (4)
6.1955	- 2.92	- 6.49 (-2)	- 1.05 (4)	1.52	- 1.22
6.1975	- 4.28	- 5.36 (0)	- 1.21	1.55	- 1.39
6.1990	- 5.49	- 9.82	- 1.32	1.45	- 1.52
6.2005	- 6.92	- 1.47 (1)	- 1.42	1.18	- 1.63
6.2015	- 7.98	- 1.82	- 1.47	8.98 (5)	- 1.69
6.2025	- 9.15	- 2.19	- 1.51	5.50	- 1.74
6.2035	- 1.04 (5)	- 2.57	- 1.53	1.55	- 1.76
6.2040	- 1.11	- 2.77	- 1.53	- 5.17 (4)	- 1.76

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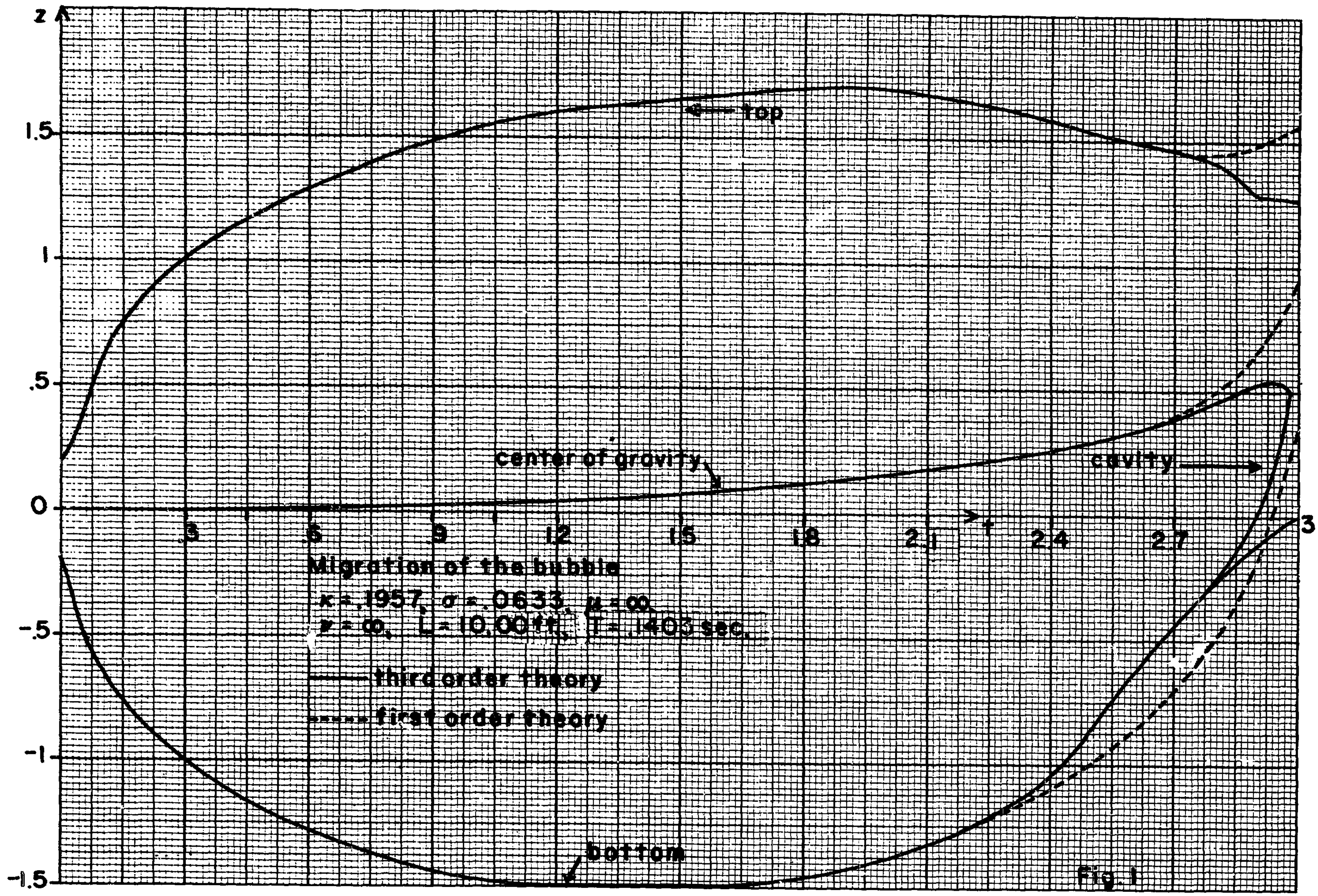
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0
-.5
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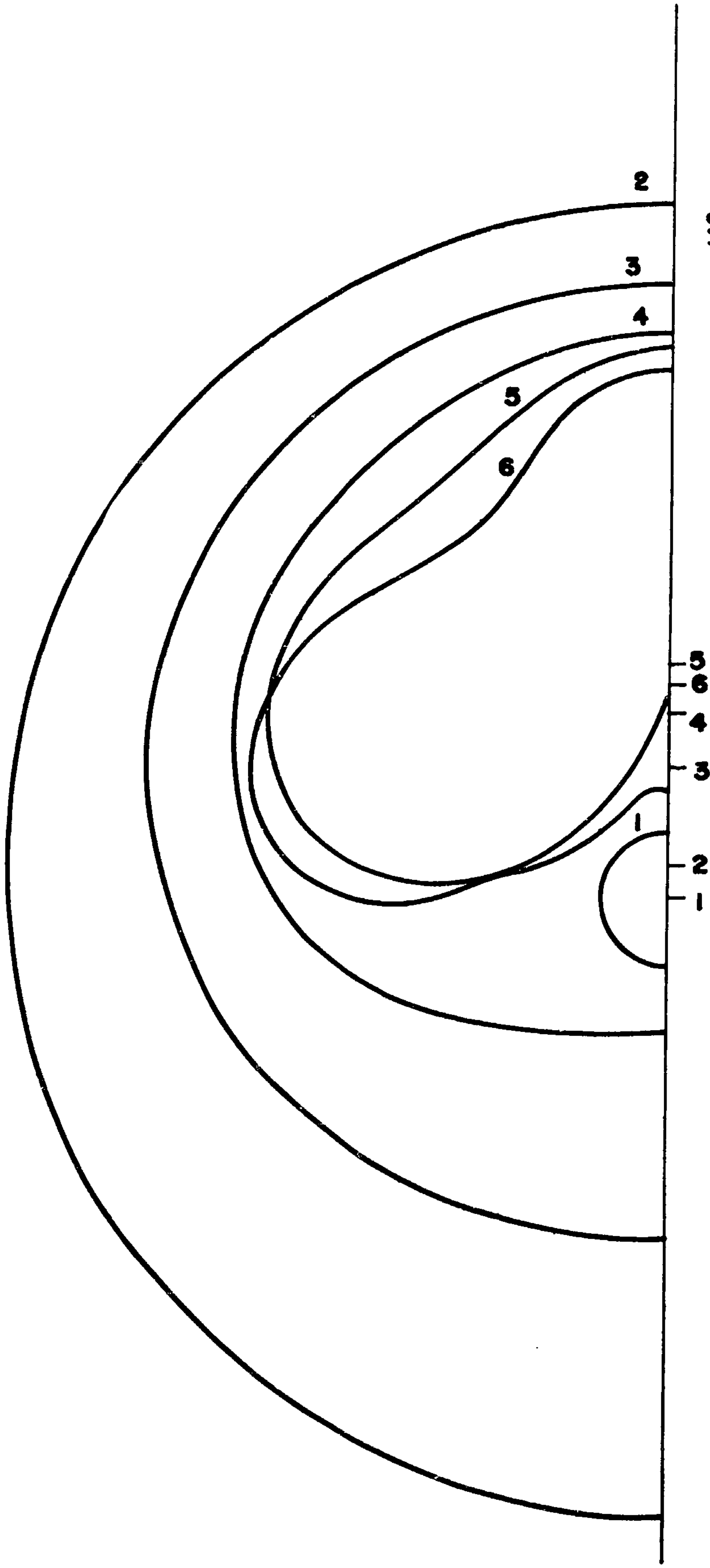
-1.5
.3
.6
.9
1.2
1.5
1.8
2.1
t
2.4
2.7
3
top
center of gravity
cavity
bottom
Migration of the bubble
κ = .1957, σ = .0633, μ = ∞,
ν = ∞, L = 10.00 ft, T = .1403 sec.
third order theory
first order theory

Fig. 1



Shape of the bubble

$\kappa = .1957$, $\sigma = .0633$,

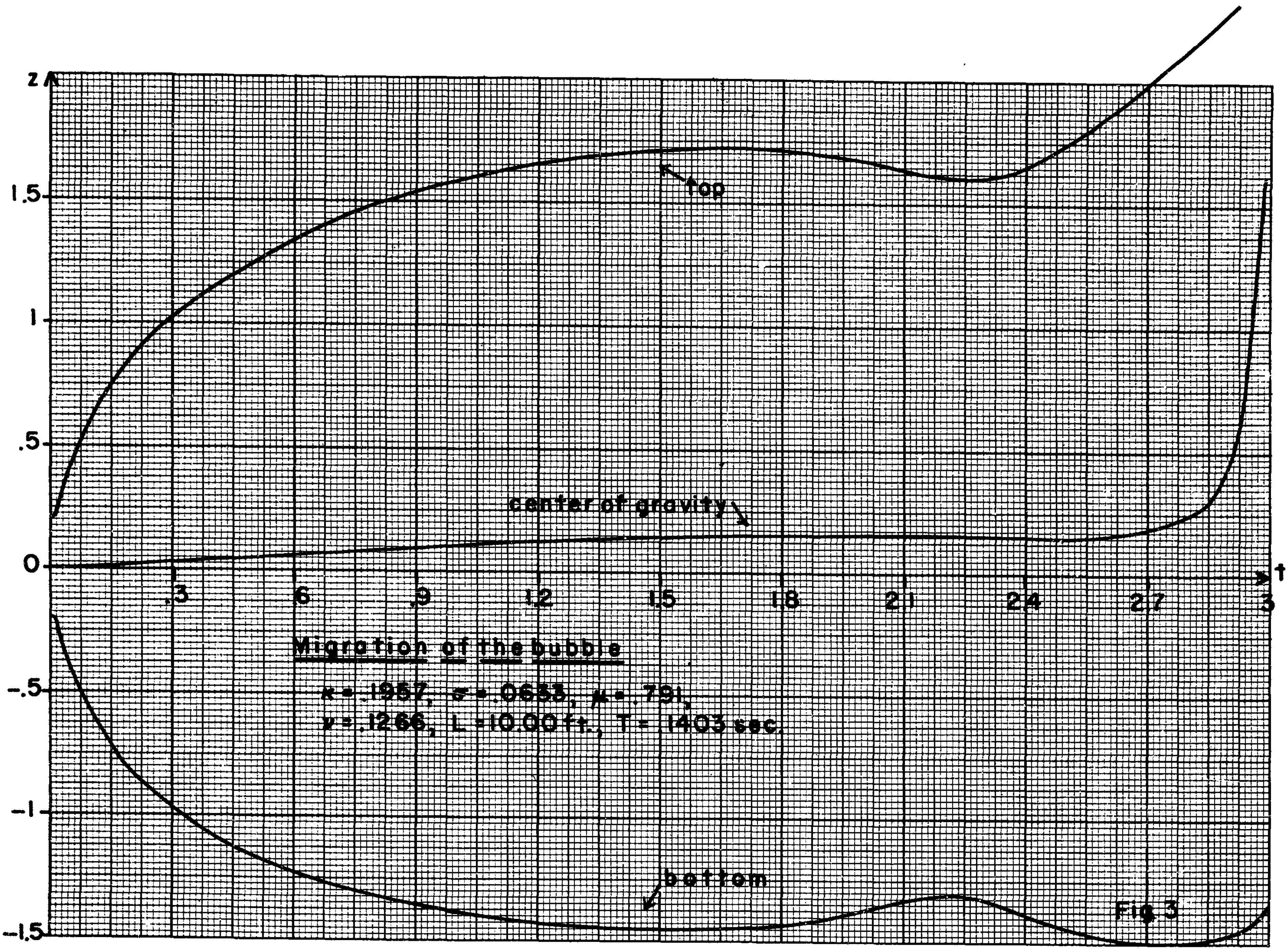
$\mu = \infty$, $\nu = \infty$,

L = 10.00 ft., T = .1403 sec.

Scale: 2.5" = 1

drawing	t	b
1	0	0
2	1.55	.080
3	2.59	.322
4	2.82	.456
5	2.94	.532
6	2.977	.514

Fig. 2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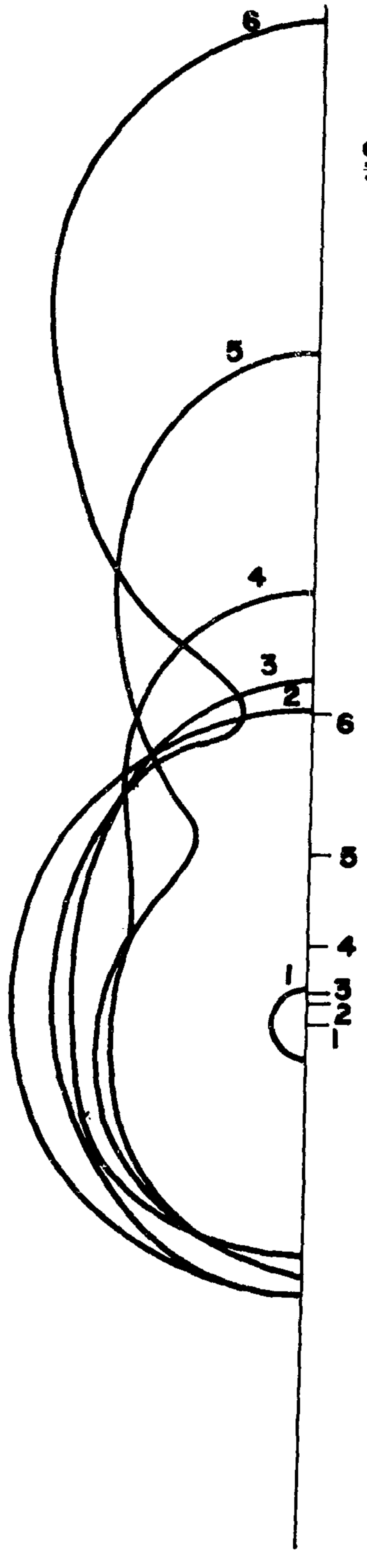
-1.5
.3
.6
.9
1.2
1.5
1.8
2.1
2.4
2.7
3
top
center of gravity
bottom
Migration of the bubble
κ = .1957, σ = .0653, μ = .791,
ν = .1266, L = 10.00 ft., T = .1403 sec.

Fig 3



Shape of the bubbe

$\kappa = .1957, \quad \sigma = .0633,$

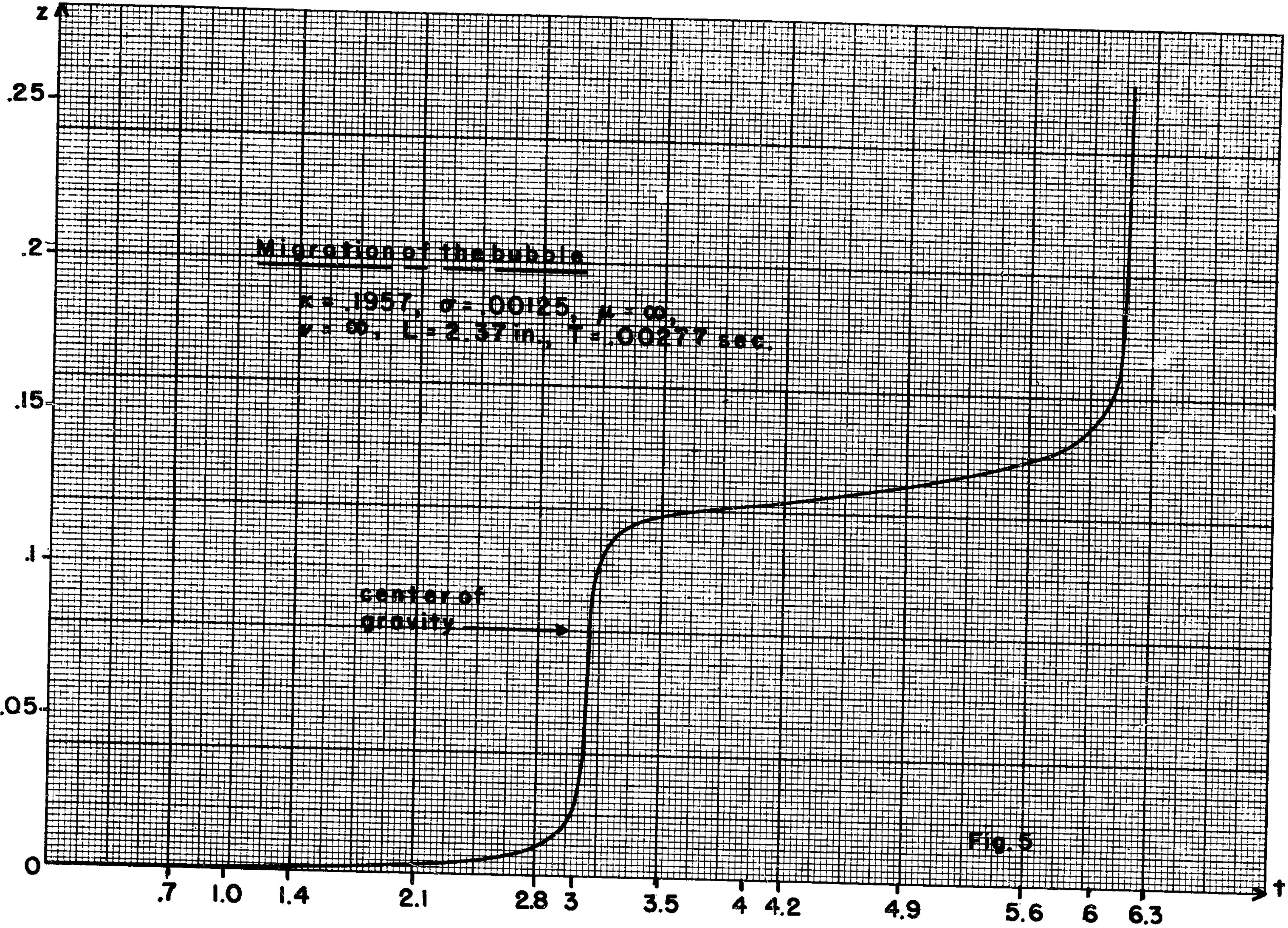
$\mu = .791, \quad \nu = .1266$

L = 10.00 ft., T = .1403 sec.

Scale: 1" = 1

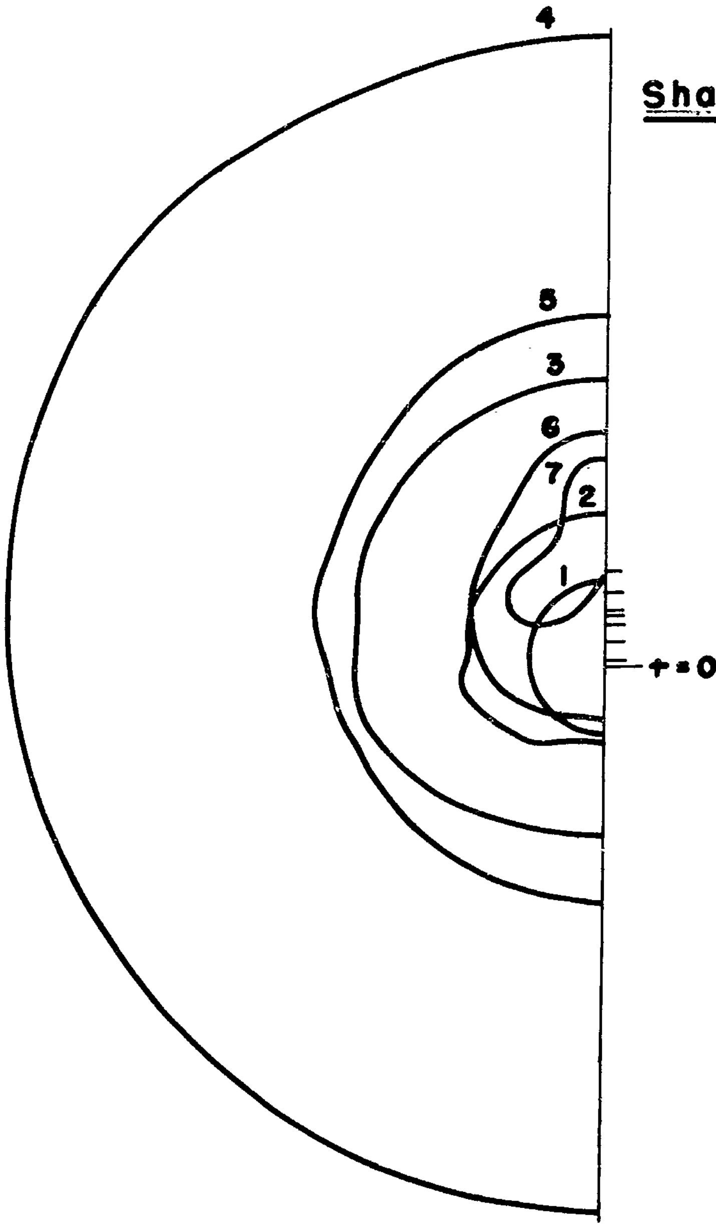
drawing	t	b
1	0	0
2	1.55	.133
3	2.59	.163
4	2.82	.408
5	2.94	.900
6	2.982	1.610

Fig. 4



Migration of the bubble
κ = .1957, σ = .00125, μ = ∞,
ν = ∞, L = 2.37 in., T = .00277 sec.
center of gravity
z
.25
.2
.15
.1
.05
0
.7
1.0
1.4
2.1
2.8
3
3.5
4
4.2
4.9
5.6
6
6.3
t

Fig. 5



Shape of the bubble

$\kappa = .1957, \quad \sigma = .00125,$

$\mu = \infty, \quad \nu = \infty,$

L = 2.37 in., T = .00277 sec.

Scale: 2" = 1

drawing	t
1	3.095
2	3.135
3	3.221
4	4.653
5	6.045
6	6.172
7	6.200

Fig. 6

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